PRACTICE MIDTERM 2 (BORCHERDS) - ANSWER KEY

PEYAM RYAN TABRIZIAN

(1) $x = \pm\frac{\pi}{3} + 2\pi n$ (n integer), basically $\cos(x) = \frac{1}{2}$

(2) $y = 30x + 1$

(3) $y' = \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2}$

(4) $y''' = \frac{-24}{(2x-1)^4}$

(5) $y' = \frac{1}{\ln(\ln(\ln(x)))} \times \frac{1}{\ln(\ln(x))} \times \frac{1}{\ln(x)} \times \frac{1}{x}$

(6) $y' = \cosh(x)\tanh(x) + \sinh(x)sech^2(x) = \sinh(x)(1 + sech^2(x))$

(7) $L(x) = 10 + \frac{1}{20}(x - 100)$, so $\sqrt{99.8} \approx L(99.8) = 10 + \frac{-0.2}{20} = 10 - 0.01 = 9.99$

(8) Candidates: $f(0) = 1$, $f(3) = 19$, $f(1) = -1$ (critical points are ± 1, but $-1 \notin [0,3]$). Absolute maximum $f(3) = 19$, Absolute minimum $f(1) = -1$.

(9) 0 (f' not defined), $\frac{1}{8}$ ($f' = 0$)

(10) f is differentiable on $[0,4]$ and $f(0) = f(4) = 1$; $c = 2$

(11) $\nearrow$ on $(-\infty, 0) \cup (2, \infty)$, $\searrow$ on $(0, 2)$. Local max $f(0) = 0$, Local min $f(2) = 4e^2$

(12) $\frac{1}{2}$ (l'Hopital's rule twice)

(13) 0 (trick question! $\frac{0}{1}$ is **NOT** an indeterminate form!)

(14) For this question, everything will be assumed **up to multiples of** 2π:
- Domain = $[0, \pi) \cup (\pi, 2\pi]$
- No local max/min
- Increasing on $(0, \pi)$ and on $(\pi, 2\pi)$ ($f'(x) = \frac{1+\cos(x)}{(1+\cos(x))^2} = \frac{1}{1+\cos(x)}$)
- Zeros = $0, 2\pi$
- No behavior at $\pm\infty$ (periodic function!)
- $f(0) = 0$

(15) Turn page for (15)

Date: Thursday, March 24th, 2011.

(15)
- Domain = $(0, \infty)$
- Local max = $f(e) = e^{\frac{1}{e}}$, No local min
- Increasing on $(0, e)$, decreasing on (e, ∞) ($f'(x) = x^{\frac{1}{x}} \left(\frac{1-\ln(x)}{x^2}\right)$)
- No zeros
- Horizontal Asymptote $y = 1$ at ∞ (take $\ln$ and use l'Hopital's rule)
- No vertical asymptotes ($\lim_{x \to 0^+} f(x) = 0^{\infty} = 0$)